$$\int \left(1+\frac{1}{t}\right)^3 \frac{1}{t^2}\,dt = \int u^3 \cdot (-du)$$

8.

$u = 1 + \frac{1}{t} = 1 + t^{-1}$

$du = -1t^{-2}\,dt$

$-du = \frac{\cancel{-}1}{t^2}\,dt$

$= -1\int u^3\,du$

$= \frac{-1u^4}{4} + C$

$= \frac{-1(1+t^{-1})^4}{4} + C$

12. $\int \frac{2x-1}{\sqrt{x+3}}dx = \int \frac{2(u-3)-1}{u^{1/2}}\,du$

$u = x+3 \rightarrow x = u-3$

$du = dx$

$$\int \frac{2u-6-1}{u^{1/2}}\,du$$

$$= \int \frac{2u-7}{u^{1/2}}\,du$$

$$= \int \left(\frac{2u^1}{u^{1/2}} - \frac{7}{u^{1/2}}\right) du$$

$$\int \left(2u^{1/2} - 7u^{-1/2}\right) du$$

$$\frac{2}{3}\cdot 2u^{3/2} - 2\cdot 7u^{1/2} + C$$

$$\frac{4}{3}u^{3/2} - 14u^{1/2} + C$$

$$\frac{4}{3}(x+3)^{3/2} - 14\sqrt{x+3} + C$$

$$\int \sin 2x \, dx = \int \sin u \cdot \frac{du}{2}$$

$u = 2x$

$du = 2\,dx$

$\frac{du}{2} = dx$

$$= \frac{1}{2}\int \sin u \, du$$

$$= -\frac{1}{2}\cos u + C$$

$$= -\frac{1}{2}\cos 2x + C$$

Check

$$\cancel{-}\frac{1}{\cancel{2}}(\cancel{-}\sin 2x)\cdot \cancel{2}$$

$$\int \csc^2\left(\frac{x}{2}\right) dx = \int \csc^2 u \cdot 2\,du$$

$$u = \frac{x}{2}$$

$$du = \frac{1}{2}\,dx$$

$$2\,du = dx$$

$$= 2\int \csc^2 u\,du$$

$$= -2\cot u + C$$

$$y = -2\cot\left(\frac{x}{2}\right) + C$$

$$\int \frac{\sin x}{\cos^2 x}\,dx = \int \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}\,dx$$

$$\int \tan x \sec x \,dx$$

$$y = \sec x + C$$

$$\int \frac{\sin x}{\cos^2 x}\,dx = \int \frac{1}{u^2}\,-du$$

$u = \cos x$

$du = -\sin x\,dx$

$-du = \sin x\,dx$

$$-\int u^{-2}\,du$$

$$\cancel{-}\frac{u^{-1}}{\cancel{-1}} + C$$

$$\frac{1}{u} + C$$

$$\frac{1}{\cos x} + C$$

$$y = \sec x + C$$

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx = \int \sin u \cdot 2\,du$$

$$u = x^{1/2} \qquad = 2\int \sin u\,du$$

$$du = \frac{1}{2}x^{-1/2}\,dx \qquad = 2(-\cos u) + C$$

$$2\,du = \frac{1}{\sqrt{x}}\,dx \qquad y = -2\cos\sqrt{x} + C$$

$$\int \underline{\underline{r^2}} \sec^2 r^3 \, \underline{\underline{dr}} = \int \sec^2 u \cdot \frac{du}{3}$$

$$u = r^3$$

$$du = 3r^2 \, dr$$

$$\frac{du}{3} = r^2 \, dr$$

$$= \frac{1}{3} \int \left(\sec^2 u\right) du$$

$$= \frac{1}{3} \tan u + C$$

$$= \frac{1}{3} \tan\left(r^3\right) + C$$

Check

$$\frac{1}{\cancel{3}} \sec^2\left(r^3\right) \cdot \cancel{3} r^2$$